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## Proximity Without Consensus in Online Multiagent Optimization

### Abstract:

We consider stochastic optimization problems in multiagent settings, where a network of agents aims to learn parameters that are optimal in terms of a global convex objective, while giving preference to locally observed streaming information. To do so, we depart from the canonical decentralized optimization framework where agreement constraints are enforced, and instead formulate a problem where each agent minimizes a global convex objective while enforcing network proximity constraints. This formulation includes online consensus optimization as a special case, but allows for the more general hypothesis that there is data heterogeneity across the network. To solve this problem, we propose using a stochastic saddle point algorithm inspired by Arrow and Hurwicz. This method yields a decentralized algorithm for processing observations sequentially received at each node of the network. Using Lagrange multipliers to penalize the discrepancy between them, only neighboring nodes exchange model information. We establish that under a constant step-size regime the time-average suboptimality and constraint violation are contained in a neighborhood whose radius vanishes with increasing number of iterations. As a consequence, we prove that the time-average primal vectors converge to the optimal objective while satisfying the network proximity constraints. We apply this method to the problem of sequentially estimating a correlated random field in a sensor network, as well as an online source localization problem, both

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of which demonstrate the empirical validity of the aforementioned convergence results.

